

Rules of Logical Inference
Outline prepared and written by:

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List of Symbols

- \rightarrow / \supset **Conditional**, if...then
 \bullet / \wedge **Conjunction**, “and, but, yet, still, however, moreover”
 \leftrightarrow / \equiv **Biconditional**: abbreviated as iff, meaning: “if and only if”
 \vee **Disjunct**: “either...or”
 \sim / \neg **Negation**: “not”
 \therefore **Conclusion**: “therefore”

1. **Modus Ponens** (M.P)

$P \rightarrow Q$
 P
 $\therefore Q$

How it is Read:

“If P, then Q”
“P”
“Therefore, Q”

2. **Modus Tollens** (M.T.)

$P \rightarrow Q$
 $\sim Q$
 $\therefore \sim P$

How it is Read:

“If P, then Q”
“not Q”
“Therefore, not P”

3. **Hypothetical Syllogism** (H.S.)

$P \rightarrow Q$
 $Q \rightarrow R$
 $\therefore P \rightarrow R$

How it is Read:

“If P, the Q”
“If Q, then R”
“Therefore, if P, then R”

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4. **Disjunctive Syllogism**: (D.S.)

$P \vee Q$
 $\sim P$
 $\therefore Q$

How it is Read:

“P or Q”
“not P”
“Therefore, Q”

5. **Constructive Dilemma** (C.D.)

$(P \rightarrow Q) \bullet (R \rightarrow S)$
 $P \vee R$
 $\therefore Q \vee S$

How it is Read:

“If P then Q, and, if R then S”
“P or R”
“Therefore, Q or S”

6. **Destructive Dilemma**: (D.D.)

$(P \rightarrow Q) \bullet (R \rightarrow S)$
 $\sim Q \vee \sim S$
 $\therefore \sim P \vee \sim R$

How it is Read:

“If P then Q, and, if R then S”
“not Q or not S”
“Therefore, not P or not R”

7. **Simplification** (Simp.)

$P \bullet Q$
 $\therefore P$

How it is Read:

“P and Q”
“Therefore, P”

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8. **Conjunction** (Conj.)

P

Q

$\therefore P \bullet Q$

How it is Read:

“P”

“Q”

“Therefore, P and Q”

9. **Addition** (Add.)

P

$\therefore P \vee Q$

How it is Read:

“P”

“Therefore, P or Q”

Replacement Rules:

10. **De Morgan's Theorems** (De M.):

$\sim(P \bullet Q) \leftrightarrow (\sim P \vee \sim Q)$

$\sim(P \vee Q) \leftrightarrow (\sim P \bullet \sim Q)$

How it is Read:

“not P and Q, if and only if, not P or not Q”

“not P or Q if and only if, not P and not Q”

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11. **Commutation** (Com):

$$(P \vee Q) \leftrightarrow (Q \vee P)$$

$$(P \bullet Q) \leftrightarrow (Q \bullet P)$$

How it is Read:

“P or Q if and only if Q or P”

“P and Q if and only if Q and P”

12. **Association** (Assoc.)

$$[P \vee (Q \vee R) \leftrightarrow (P \vee Q) \vee R]$$

$$[P \bullet (Q \bullet R) \leftrightarrow (P \bullet Q) \bullet R]$$

How it is Read:

“P, or Q or R, if and only if P or Q, or R”

“P, and Q or R, if and only if, P and Q, and R”

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13. **Distribution** (Dist.):

$$[P \bullet (Q \vee R)] \leftrightarrow [(P \bullet Q) \vee (P \bullet R)]$$

$$[P \vee (Q \bullet R)] \leftrightarrow [(P \vee Q) \bullet (P \vee R)]$$

How it is Read:

“P and, Q or R, if and only if P and Q, or P and R”

“P or, Q and R, if and only if P or Q, and P or R”

14. **Double Negation** (D.N.):

$$P \leftrightarrow \sim \sim P$$

How it is Read:

“P if and only if not, not P”

15. **Transportation** (Trans.):

$$(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$$

How it is Read:

“If P then Q, if and only if, if not Q then not P”

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16. **Material Implication** (Impl):
 $(P \rightarrow Q) \leftrightarrow (\sim P \vee Q)$

How it is Read:

“If P then Q, if and only if not P or Q”

17. **Material Equivalence** (Equiv):
 $(P \leftrightarrow Q) \leftrightarrow [(P \rightarrow Q) \bullet (Q \rightarrow P)]$
 $(P \leftrightarrow Q) \leftrightarrow [(P \bullet Q) \vee (\sim P \bullet \sim Q)]$

How it is Read:

“P if and only if Q, if and only if, if P then Q and if Q then P”
“P if and only if Q, if and only if, P and Q or not P and not Q”

18. **Exportation** (Exp.):
 $[(P \bullet Q) \rightarrow R] \leftrightarrow [P \rightarrow (Q \rightarrow R)]$

How it is Read:

“If P and Q, then R, if and only if, if P, then if Q then R”

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19. **Tautology** (Taut):

($P \vee P$)

($P \bullet P$)

How it is Read:

“P or P”

“P and P”