

# Critical Thinking

Outline prepared and written by:

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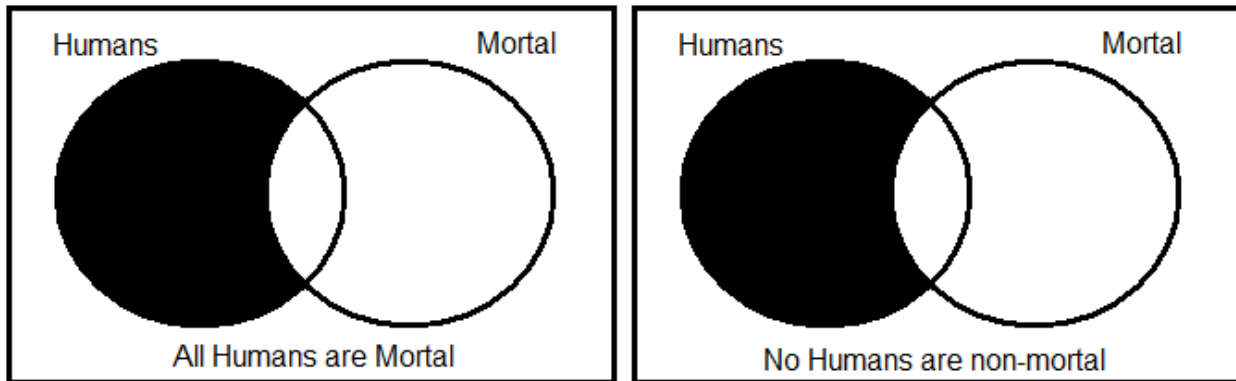
<http://jasonjcampbell.org/home.php>

Youtube Playlist Link:

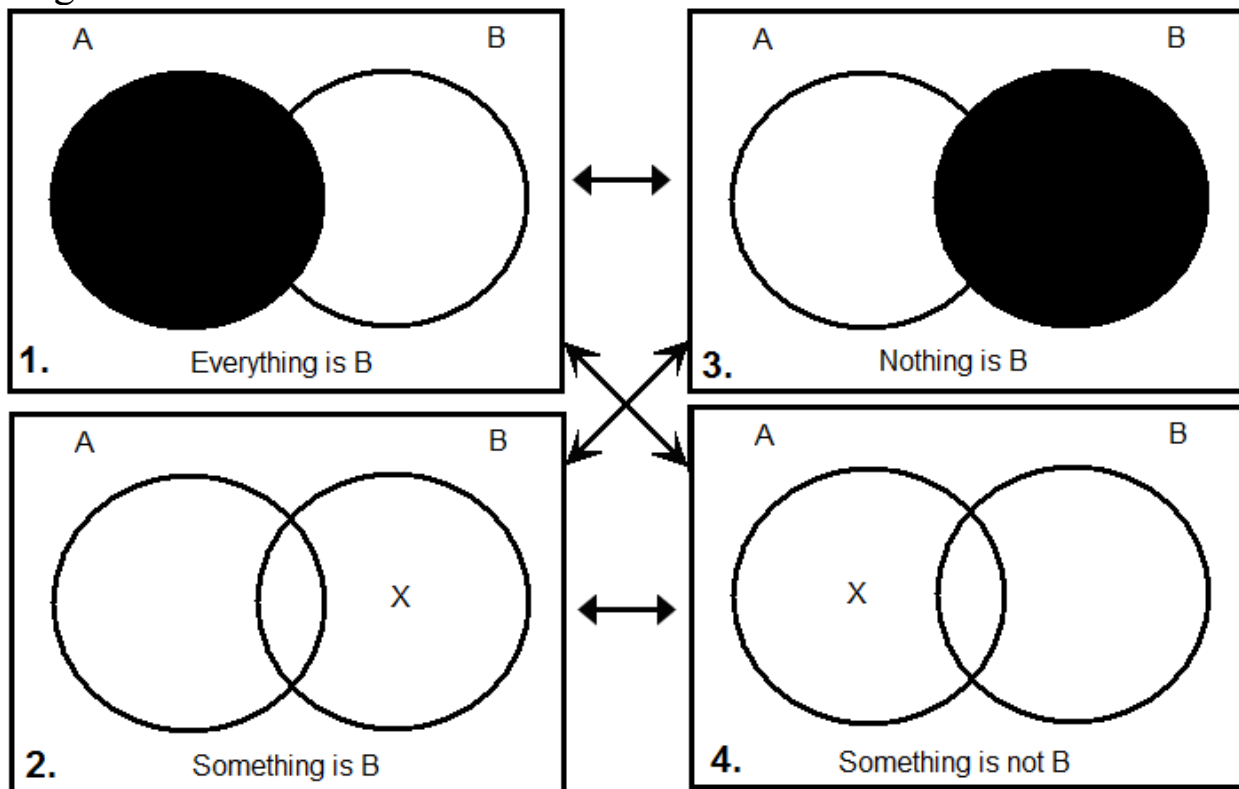
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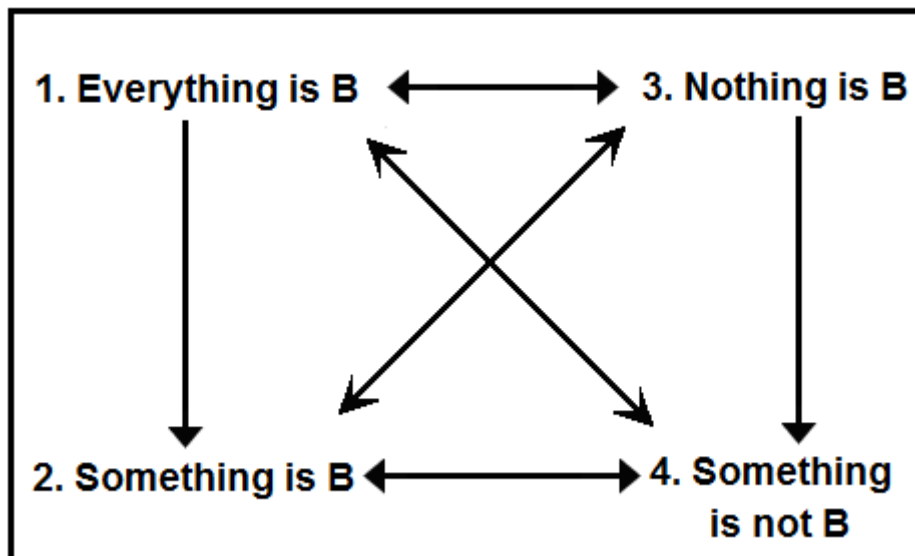
## §1:0 **Contraries, Contradictories, Subcontraries and Subalternation** [[link](#)]

- Please Review the Notes [[here](#)] and the corresponding video [[here](#)]



The statement "**All humans are mortal**" is equivalent to the statement "**no humans are non-mortal**" because they share the same Venn diagram.





**Logical Implications:**<sup>1</sup>

**Contradictories:** One must be True the other must be False

1. If 1. is True then 4. has to be False. [Both cannot be T and F]
2. If 4. is True then 1. has to be False. [Both cannot be T and F]
3. If 2. is True then 3. has to be False. [Both cannot be T and F]
4. If 3. is True then 2. has to be False. [Both cannot be T and F]

**Contraries:**

5. 1 and 3 can both be False but not True: if something is B

**Subcontraries:**

6. 2 and 4 can both be True but not False:

**Subalternation:**

7. If 1. is True 2. must be True. But the Truth of 2. doesn't imply the Truth of 1. Furthermore, you cannot assert the Truth of 1. From the Truth of 2. [Cannot universally generalize from an Existential instantiation].

8. If 3. is True 4. must be True. But the Truth of 4. doesn't imply the Truth of 3. Furthermore, you cannot assert the Truth of 3. From the Truth of 4.

1. Two Classes of Universal Propositions / Negated Existential

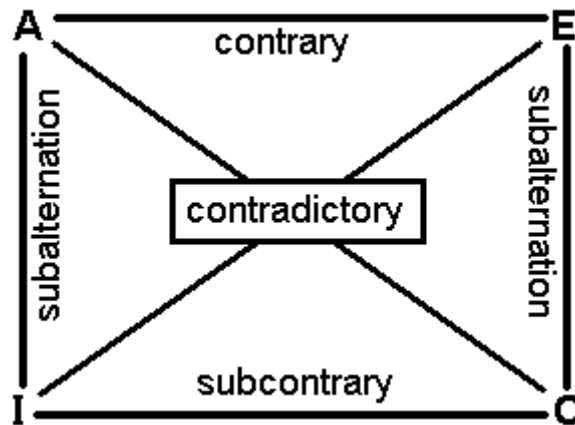
- i. **A** Propositions: Universal Affirmation
- ii. **E** Propositions: Universal Negation

2. Two Classes of Existential Propositions

- i. **I** Propositions: Existential Affirmation
- ii. **O** Propositions: Existential Negation

<sup>1</sup> Jason, Gary. *Critical Thinking: Developing an Effective Worldview*. Belmont: Wadsworth, 2001. p. 223.

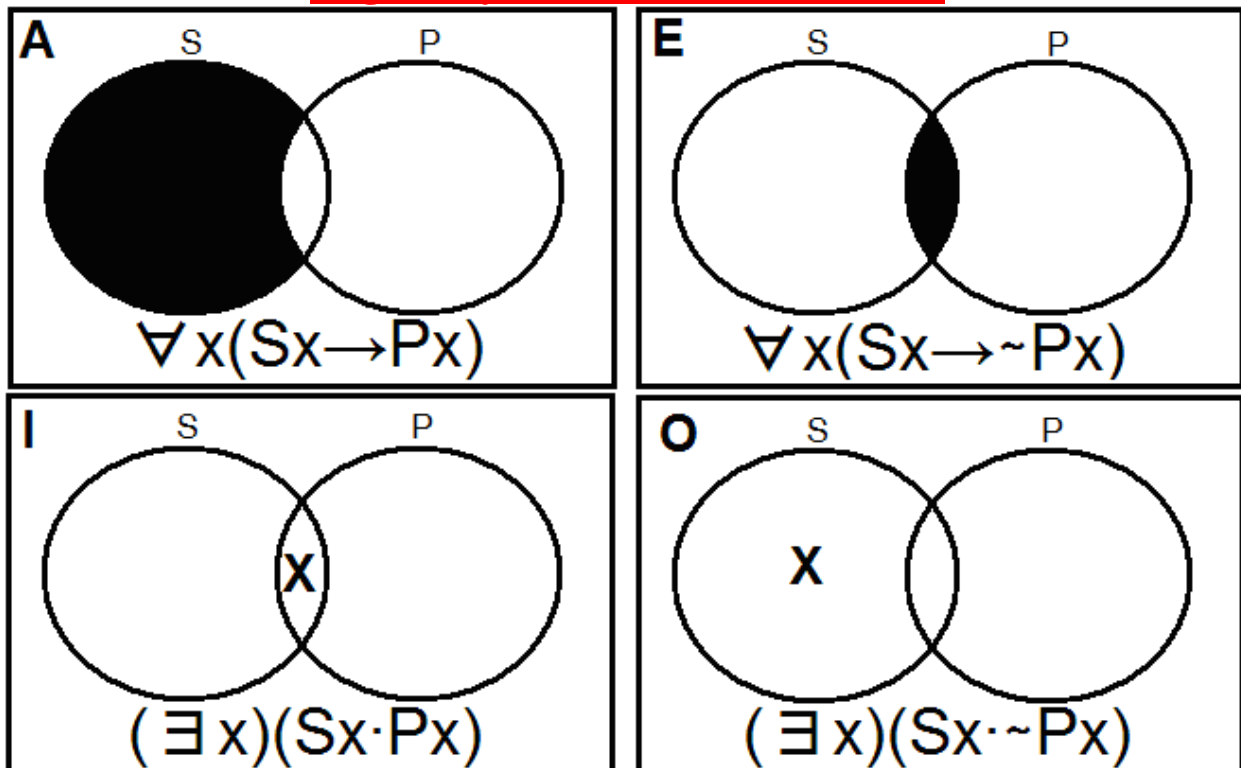
**Logical Relations of the Four Classes:**



[\[image reference\]](#)

**A:** The universal affirmation is contrary to the universal negation, contradictory to the existential negation and superaltern to the existential affirmation. **E:** The universal negation is contrary to the universal affirmation, contradictory to the existential affirmation and superaltern to the existential negation. **I:** The existential affirmation is subcontrary to the existential negation, contradictory to the universal negation and subaltern to the universal affirmation. **O:** The existential negation is subcontrary to the existential affirmation, contradictory to the universal affirmation and subaltern to the universal negation.

**Logical Symbolization of Relations**



- A:**  $\forall x (Sx \rightarrow Px)$  is contrary to  $\forall x (Sx \rightarrow \sim Px)$ , contradictory to  $(\exists x) (Sx \cdot \sim Px)$  and superaltern to  $(\exists x) (Sx \cdot Px)$ .
- E:**  $\forall x (Sx \rightarrow \sim Px)$  is contrary to  $\forall x (Sx \rightarrow Px)$ , contradictory to  $(\exists x) (Sx \cdot Px)$  and superaltern to  $(\exists x) (Sx \cdot \sim Px)$ .
- I:**  $(\exists x) (Sx \cdot Px)$  is subcontrary to  $(\exists x) (Sx \cdot \sim Px)$ , contradictory to  $\forall x (Sx \rightarrow \sim Px)$  and subaltern  $\forall x (Sx \rightarrow Px)$ .
- O:**  $(\exists x) (Sx \cdot \sim Px)$  is subcontrary to  $(\exists x) (Sx \cdot Px)$ , contradictory to  $\forall x (Sx \rightarrow Px)$  and subaltern to  $\forall x (Sx \rightarrow \sim Px)$ .

## **§2:0: Heuristic Modeling from the Three Forms of Analogies**

Heuristic Models are Explanatory Models

**The Three Forms of Analogies:**<sup>2</sup> *Assume that A is like B*

1. **Positive** analogy: A and B **share** properties  $P_1, P_2, P_3, \dots$
2. **Negative** analogy: A and B **do not share** properties  $N_1, N_2, N_3, \dots$
3. **Neutral** analogy: A and B **may or may not share** properties  $O_1, O_2, O_3, \dots$

**[Example 1]** analogical

"Robert quickly established dominance like a Silverback Gorilla."

1. **positive analogy:** [Shared Properties]: Robert and the Silverback Gorilla both established dominance.
2. **negative analogy:** [Properties not shared]: they are different species, different sizes, with different levels of intelligence.
3. **neutral analogy:** [Properties that may or may not be shared]: the precise way in which dominance was established.

**[Example 2]**

R = Random Property Generator. Generates  $P_1, P_2, P_3, R$

$\alpha$		$\beta$		$\gamma$		$\delta$	
P1		P1			P1		R
P2		P2	P3	P3			R

<sup>2</sup> Jason, Gary. *Critical Thinking: Developing an Effective Worldview*. Belmont: Wadsworth, 2001. p. 223. Additional resource [[HERE](#)].

$\alpha$		$\beta$		$\gamma$		$\delta$	
P1		P1			P1		R
P2		P2	P3	P3			R

**Assessing Analogical Relations:**

**Alpha:**

1.  $\alpha$  has a positive analogical relationship with  $\beta$  and  $\gamma$  w/ respect to P1
2.  $\alpha$  has a positive analogical relationship with  $\beta$  w/ respect to P2
3.  $\alpha$  has a negative analogical relationship with  $\beta$  and  $\gamma$  w/ respect to P3
4.  $\alpha$  has a neutral analogical relationship with  $\delta$  w/ respect to R

**Critical Thinking Questions:**

Assume that adaptability **increases** as the number of positive analogical relations **increase**

1. If adaptability increases as the number of positive analogical relations increase, which group is the most adaptable and why?
2. In an act of substitution, where properties are replaced, can adaptability **decrease** with a **decrease** in negative analogical relations? why?
3. In an act of substitution, where properties are replaced, can adaptability **decrease** with an **increase** in negative analogical relations? why?
4. If adaptability decreases as the number of negative analogical relations increase, which group is the least adaptable and why?

**Meta-Properties:** Properties of Properties: Now emphasize location.

Quadrant		$\gamma$		$\delta$	
1.	2.	P1	P1	R	P3
3.	4.	P1	P1	P1	P1

**Quadrants:** quadrant location is a **primary property** of the group [explain] fixed-inherency.

Example of this concept: [http://www.youtube.com/watch?v=UOKMVTfictw&feature=channel\\_video\\_title](http://www.youtube.com/watch?v=UOKMVTfictw&feature=channel_video_title)

**Properties:** P1, P2, P3 and R are **secondary properties**, i.e., attributable to the group.

**Gamma contains:** P1.1, P1.2, P1.3, P1.4

**Delta contains:** R.1, P3.2, P1.3, P1.4

**Analogical Relations with Meta-Properties:**

$\alpha$		$\beta$		$\gamma$		$\delta$	
P1		P1			P1		R
P2		P2	P3	P3			R

**Alpha:**

1.  $\alpha$  has a positive analogical relationship with  $\beta$  and  $\gamma$  w/ respect to P1 and  $\beta$  w/ respect to P1.1
2.  $\alpha$  has a positive analogical relationship with  $\beta$  w/ respect to P2.3
3.  $\alpha$  has a negative analogical relationship with  $\beta$  and  $\gamma$  w/ respect to P3
4.  $\alpha$  has a neutral analogical relationship with  $\delta$  w/ respect to R

**Using Heuristic Modeling to Determine Analogical Meta-Properties**

**GAME RULES:** Given the following descriptions, determine the properties of the following 4 Groups:  $\alpha$   $\beta$   $\gamma$   $\delta$ . The random property generator [R] cannot occupy the first quadrant of any group except  $\delta$ . The random property generator is the only property allowed to be replicated within any group. The only properties available are: P1, P2, P3, P4, R.

**\*\*DESCRIPTION\*\***

$\alpha$  has a positive analogical relationship with  $\beta$  w/ respect to P4.4 and a negative analogical relationship with  $\gamma$   $\delta$  with respect to P4.  $\beta$  has a negative analogical relationship with  $\alpha$   $\gamma$   $\delta$  with respect to P1.  $\beta$   $\gamma$   $\delta$  have a positive analogical relationship with respect to P3.3.  $\beta$  has a positive analogical relationship with  $\alpha$  w/ respect to P2 and a negative analogical relation with  $\gamma$   $\delta$  w/ respect to P2.  $\alpha$   $\beta$  have a positive analogical relationship with  $\gamma$  with respect to R.

$\alpha$		$\beta$		$\gamma$		$\delta$	
P2				P1			
P1							P1

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### §3:0: Interwoven Arguments<sup>3</sup>

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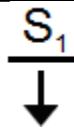
#### Two Forms of Premise Support:

1. **Single support**: an instance in which **one premise** supports a conclusion.

i. e.g., "Jason is a professor and researcher; therefore, he is a researcher."

ii. e.g., "All squares have interior angles totaling  $360^0$ , so if this is a square it has interior angles totaling  $360^0$ ."

#### Single Support Argument

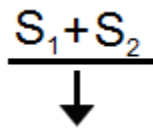


[Where  $S$  stands for any arbitrary statement]

2. **Linked support**: "the support of a conclusion by **two or more premises** that work together" (p. 59). Premises in a linked supported chain are  $P_1, P_2, \dots, P_n$

i. e.g., All rationalists are idealists. All idealists deny mind independent reality; therefore, all rationalist deny mind independent reality.

#### Linked Support Argument



[Where  $S$  stands for any arbitrary statement]

**Enthymemes**: a syllogism in which one of the premises is missing or implicit.

i. e.g., **Enthymeme**: "If it rains outside I'll bring my umbrella. Therefore, if it rains outside I won't get **wet**." [need a premise connecting "wet" with "umbrella"]

ii. e.g., **The Full Argument**: "If it rains outside, I'll bring my umbrella. If I bring my umbrella, I won't get wet. Therefore, if it rains outside, I won't get wet." [ref]

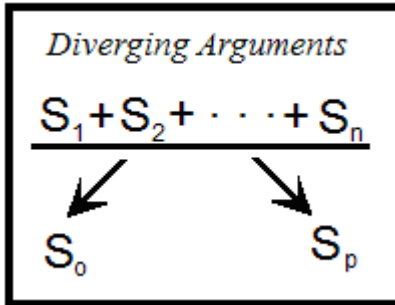
**Premise Cluster**: "any group of mutually linked premises (including any one single-supported premise)" (p. 60).

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<sup>3</sup> <http://puffin.creighton.edu/yuan/logic/AnCh1.4.-Numbering.htm>

#### **4 Forms of Interwoven Arguments:**<sup>4</sup>

1. **Diverging Arguments:** "begin at the same premise cluster but draw out different and quite often complementary conclusions" p.60.



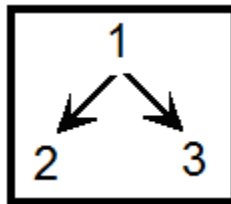
"The American dollar is very strong person with the European currencies. This means that European goods here will be real bargains, but it also means that our exports abroad will be at a competitive disadvantage." p.60.

1= the American dollar is strong in comparison with the European currencies.

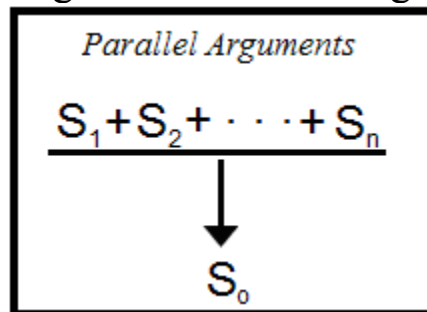
2= European goods here will be a bargain.

3= American goods abroad will be at a competitive disadvantage.

#### **The Representation of this Divergent Argument**



2. **Parallel Arguments:** arguments establishing contrast conclusions.



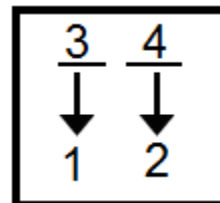
"Well I'm going to bomb physics, I'm really going to do well in chemistry. My physics grades are F, D, and D-, whereas my chem grades are A, A+ and B+." p.61.

1= I'm going to do poorly in physics.

2= I'm going to do well in chemistry.

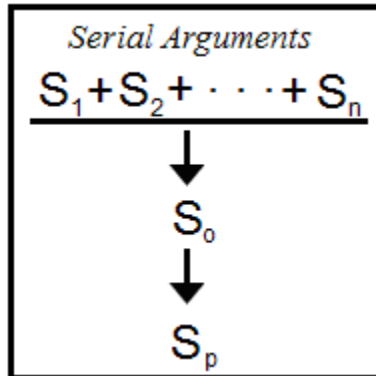
3= My physics grades are F, D, and D-

4= My chem grades are A, A+ and B+



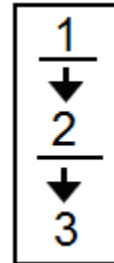
<sup>4</sup> Jason, Gary. *Critical Thinking: Developing an Effective Worldview*. Belmont: Wadsworth, 2001. p. 61.

3. **Serial Arguments:** "the conclusion of one argument becomes the premise cluster for another" (p. 61).



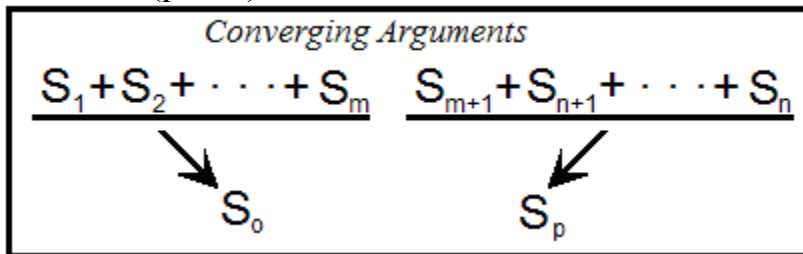
"Jazz is improvisational music. This tells us that jazz is better appreciated live than on record. But that in turn means that a true jazz lover should go to jazz nightclubs" (p. 62).

- 1= jazz is improvisational music.
- 2= jazz is better appreciated live than on record.
- 3= jazz lovers should go to jazz nightclubs.



4. **Converging Arguments:**

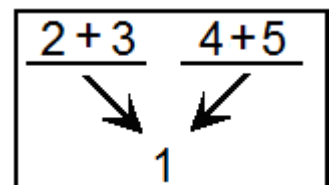
"involve the elaboration of two different lines of attack on the same ultimate conclusion" (p. 62).



*The book shows the image above, but it must be a typo. Converging arguments converge on the same statement.*

"Evidence for the existence of the great pumpkin comes from two quite different directions. First, there is the undeniable unanimity of the testimony of small children. When testimony is unanimous, it is very convincing. Second, fossils reveal pumpkin seeds the size of tennis shoes. The only way to account for such mammoth seeds is to postulate the existence of the great pumpkin" (p. 62)

- 1= the great pumpkin exists.
- 2= small children are unanimous in their testimony about the great pumpkin.
- 3= unanimous testimony is especially convincing.
- 4= fossils reveal giant pumpkin seeds.
- 5= postulating the existence of the great pumpkin is the only way to explain those seeds.

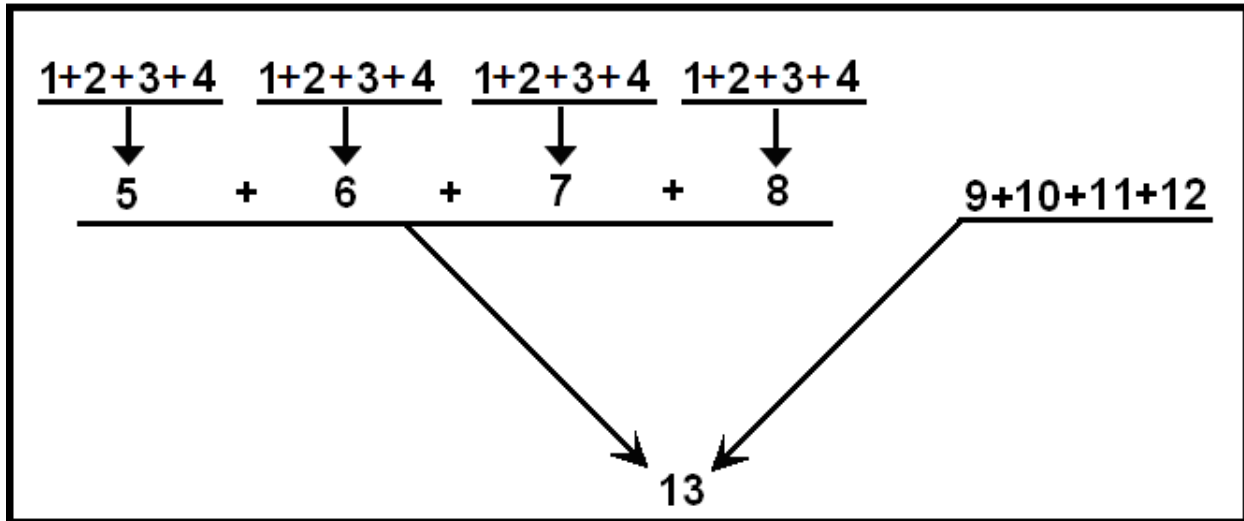


## Mapping John F. Kennedy's *Civil Rights Address*

...This Nation was founded by men of many nations and backgrounds. It was founded on the principle that all men are created equal, and that the rights of every man are diminished when the rights of one man are threatened...And when Americans are sent to Vietnam or West Berlin, we do not ask for whites only. It ought to be possible, therefore, for American students of any color to attend any public institution they select without having to be backed up by troops. It ought to be possible for American consumers of any color to receive equal service in places of public accommodation...and it ought to be possible for American citizens of any color to register and to vote in a free election without interference or fear of reprisal. It ought to be possible, in short, for every American to enjoy the privileges of being American without regard to his race or his color...But this is not the case. The Negro baby born in America today, regardless of the section of the State in which he is born, has about one-half as much chance of completing a high school as a white baby born in the same place on the same day, one-third as much chance of completing college, one-third as much chance of becoming a professional man, twice as much chance of becoming unemployed, about one-seventh as much chance of earning \$10,000 a year, a life expectancy which is 7 years shorter, and the prospects of earning only half as much. This is not a sectional issue. Difficulties over segregation and discrimination exist in every city, in every State of the Union, producing in many cities a rising tide of discontent that threatens the public safety. Nor is this a partisan issue. In a time of domestic crisis men of good will and generosity should be able to unite regardless of party or politics. This is not even a legal or legislative issue alone. It is better to settle these matters in the courts than on the streets, and new laws are needed at every level, but law alone cannot make men see right. We are confronted primarily with a moral issue. It is as old as the Scriptures and is as clear as the American Constitution. The heart of the question is whether all Americans are to be afforded equal rights and equal opportunities, whether we are going to treat our fellow Americans as we want to be treated.

1. This Nation was founded by men of many nations and backgrounds
2. It was founded on the principle that all men are created equal
3. the rights of every man are diminished when the rights of one man are threatened
4. And when Americans are sent to Vietnam...we do not ask for whites only.
5. [conclusion] It ought to be possible, therefore, for American students of any color to attend any public institution they select without having to be backed up by troops.
6. [conclusion] It ought to be possible for American consumers of any color to receive equal service in places of public accommodation.
7. [conclusion] it ought to be possible for American citizens of any color to register and to vote in a free election without interference or fear of reprisal
8. [conclusion] It ought to be possible, in short, for every American to enjoy the privileges of being American without regard to his race or his color.
9. The Negro baby born in America today, regardless of the section of the State in which he is born, has about one-half as much chance of completing a high school as a white baby born in the same place on the same day, one-third as much chance of completing college, one-third as much chance of becoming a professional man, twice as much chance of becoming unemployed, about one-seventh as much chance of earning \$10,000 a year, a life expectancy which is 7 years shorter, and the prospects of earning only half as much.
10. Difficulties over segregation and discrimination exist in every city, in every State of the Union, producing in many cities a rising tide of discontent that threatens the public safety.

11. In a time of domestic crisis men of good will and generosity should be able to unite regardless of party or politics.
12. It is better to settle these matters in the courts than on the streets, and new laws are needed at every level, but law alone cannot make men see right. We are confronted primarily with a moral issue.
13. [conclusion]... all Americans are to be afforded equal rights and equal opportunities.




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#### §4:0: Nonmonotonic Reasoning and Model Incompleteness

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#### Three Properties of First Order Predicate Logic:

[Predicate Logic Series [Here](#)]

1. "It is complete with respect to the domain of interest. In other words, all the facts that are necessary to solve the problem are present in the system or can be derived from those that are by the conventional rules of first order logic."<sup>5</sup>
2. "It is consistent" (p.197)
3. "The only way it can change is that new facts can be added as they become available. If these new facts are consistent with all the other facts that have already been asserted, then nothing will ever be retracted from the set of facts that are known to be true. This property is called *monotonicity*." (p.197)

If these three properties of first order predicate logic [FOPL] cannot be satisfied because of the nature of the problem, then obviously FOPL

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<sup>5</sup> Elaine Rich, Kevin Knight. *Artificial Intelligence; Second Edition*. New York: McGraw-Hill, 1991. pg. 197.

can't be used in attempting to solve problems that fail to satisfy any or all of the above conditions. Thus, we need a system of problem solving that can be used when any or all of these conditions are not satisfied. That system is a system of nonmonotonic logic.

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FOPL **can address** the following: [It is known that  $\sim P$ ]

FOPL **cannot address** the following: [It is not known *whether* P]

However, nonmonotonic logic [NonL] **can address** the following: [It is not known whether P]

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### **Structural Differences in Nonmonotonic Logic:**

1. **In FOPL:** "if some set of axioms  $T$  entail the truth of some statement  $w$ , then  $T$  combined with another set of axioms  $N$  also entail  $w$ " (p. 198). [explain].

For example, consider the following two **axioms**:

1. (Axiom T):  $P \rightarrow (P \cdot P)$       2. (Axiom N):  $(P \cdot Q) \rightarrow P$

Let's consider the following: (**Statement**  $w$ :  $(P \cdot P)$ )

Thus the combination of  $T$  and  $N$  preserves the truth of  $w$ , because the assumption is that  $w$  remains constant and constantly valid. In FOPL new information **cannot compromise** the validity of such statements.

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1a. **In NonL:** NonL is defeasible. "A nonmonotonic inference may be defeated (rendered invalid) **by the addition** of new information that violates assumptions that were made **during the original reasoning process**" (p. 198).

For example: Consider the following statement:

**Original reasoning process:** "Mary is a loving mother who would never harm her children."

**New Information:** "I've recently discovered that Mary was diagnosed with *Munchausen by proxy*."

**Synthesis:** The idea that Mary would never harm her children is **defeated** by her diagnosis. Thus, part of our original reasoning has been "defeated" i.e., "Mary would never harm her children" has been rendered False by her diagnosis. [Obviously this assumes accuracy in her diagnosis].

## Nonmonotonic Logic and Default Reasoning:

1. **Default Reasoning:** Pertains to conclusions that are most likely true.

[Note]: unlike predicate logic where truth is definitive, in NonL, formal systems preserve uncertainty, i.e., there are varying degrees of truth.

2. **Reading NonL Language:** the language of FOPL is "augmented with the modal operator  $M$ "<sup>6</sup> which means "**is consistent**"

All three variations are read the same

" $\forall x,y: EscalateConflict(x,y) \bullet M RefuseNegotiation(x,y) \rightarrow DeclareWar(x,y)$ "

" $\forall x,y: EscalateConflict(x,y) \wedge M RefuseNegotiation(x,y) \rightarrow DeclareWar(x,y)$ "

" $\forall x,y: EscalateConflict(x,y) \wedge : RefuseNegotiation(x,y) \rightarrow DeclareWar(x,y)$ "

[**Note:** ( $\bullet$  and  $\wedge$ ) and ( $M$  and  $:$ ) are the interchangeable]

[**This is read**]: "For all x and y, if x and y escalate the conflict and if the fact that x and y refuse negotiations **is consistent with** everything else believed to be true, then x and y will declare war."

### **Creating a Nonmonotonic Serial Argument:**

[**Remember:** in a serial argument the consequence of one argument becomes the antecedent of another]

" $\forall x,y: DeclareWar(x,y) \bullet M UseFullForce(x,y) \rightarrow MaximumCasualties(x,y)$ "

"For all x and y, if x and y declare war and if the fact that x and y use full force **is consistent with** everything else believed to be true, then x and y will experience maximum casualties."

### **Logical Consistency of Syllogism**

" $\forall x,y: EscalateConflict(x,y) \bullet M RefuseNegotiation(x,y) \rightarrow MaximumCasualties(x,y)$ "

[**This is read**]: "For all x and y, if x and y escalate the conflict and if the fact that x and y refuse negotiations **is consistent with** everything else believed to be true, then x and y will experience maximum casualties."

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<sup>6</sup> Elaine Rich, Kevin Knight. *Artificial Intelligence; Second Edition*. New York: McGraw-Hill, 1991. pg. 201.

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## §4:1: Applying Nonmonotonic Reasoning to Critical Thinking

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### *The Web of Belief*

"Let Abbott, Babbitt, and Cabot be suspects in a murder case. Abbott has an alibi, in the register of a respectable hotel in Albany. Babbitt also has an alibi, for his brother-in-law testified that Babbitt was visiting him in Brooklyn at the time. Cabot pleads alibi too, claiming to have been watching a ski meet in the Catskills, but we have only his word for that. So we believe

- (1) that Abbott did not commit the crime,
- (2) that Babbitt did not,
- (3) that Abbott or Babbitt or Cabot did.

But presently Cabot documents his alibi — he had the good luck to have been caught by television in the sidelines at the ski meet. A new belief is thus thrust upon us:

- (4) that Cabot did not.

Our beliefs [1] through (4) are inconsistent, so we must choose one for rejection. Which has the weakest evidence? The basis for (1) in the hotel register is good, since it is a fine old hotel. The basis for (2) is weaker, since Babbitt's brother-in-law might be lying. The basis for (3) is perhaps twofold: that there is no sign of burglary and that only Abbott, Babbitt, and Cabot seem to have stood to gain from the murder apart from burglary. This exclusion of burglary seems conclusive, but the other consideration does not; there could be some fourth beneficiary. For (4), finally, the basis is conclusive: the evidence from television. Thus (2) and (3) are the weak points. To resolve the inconsistency of (1) through [4] we should reject [2] or (3), thus either incriminating Babbitt or widening our net for some new suspect.

See also how the revision progresses downward. If we reject (2), we also revise our previous underlying belief, however tentative, that the brother-in-law was telling the truth and Babbitt was in Brooklyn. If instead we reject (3), we also revise our previous underlying belief that none but Abbott, Babbitt, and Cabot stood to gain from the murder apart from burglary.

Finally a certain arbitrariness should be noted in the organization of this analysis. The inconsistent beliefs (1) through (4) were singled out, and then various further beliefs were accorded a subordinate status as underlying evidence: a belief about a hotel register, a belief about the prestige of the hotel, a belief about the television, a perhaps unwarranted belief about the veracity of the brother-in-law, and so on. We could instead have listed this full dozen of beliefs on an equal footing, appreciated that they were in contradiction, and proceeded to restore consistency by weeding them out in various ways. But the organization lightened our tasks: it focus our attention on four prominent beliefs among which to drop one, and then it ranged the other beliefs under these four as mere aids to choosing which of the four to drop.

The strategy illustrated would seem in general to be a good one: divide and conquer. When a set of beliefs has accumulated to the point of contradiction, find the smallest selection of them you can that still involves contradiction; for instance, [1] through [4]. For we can be sure that we are going to have to drop some of the beliefs in that subset, whatever else we do. In reviewing and comparing the evidence for the beliefs in the subset, then, we will find ourselves led down in a rather systematic way to other beliefs of the set. Eventually we find ourselves dropping some of them too.

In probing the evidence, where do we stop? In probing the evidence for [1] through (4) we dredged up various underlying beliefs, but we could have probed further, seeking evidence in turn for them. In practice the probing stops when we are satisfied how best to restore consistency: which ones to discard among the beliefs we have canvassed.<sup>7</sup>

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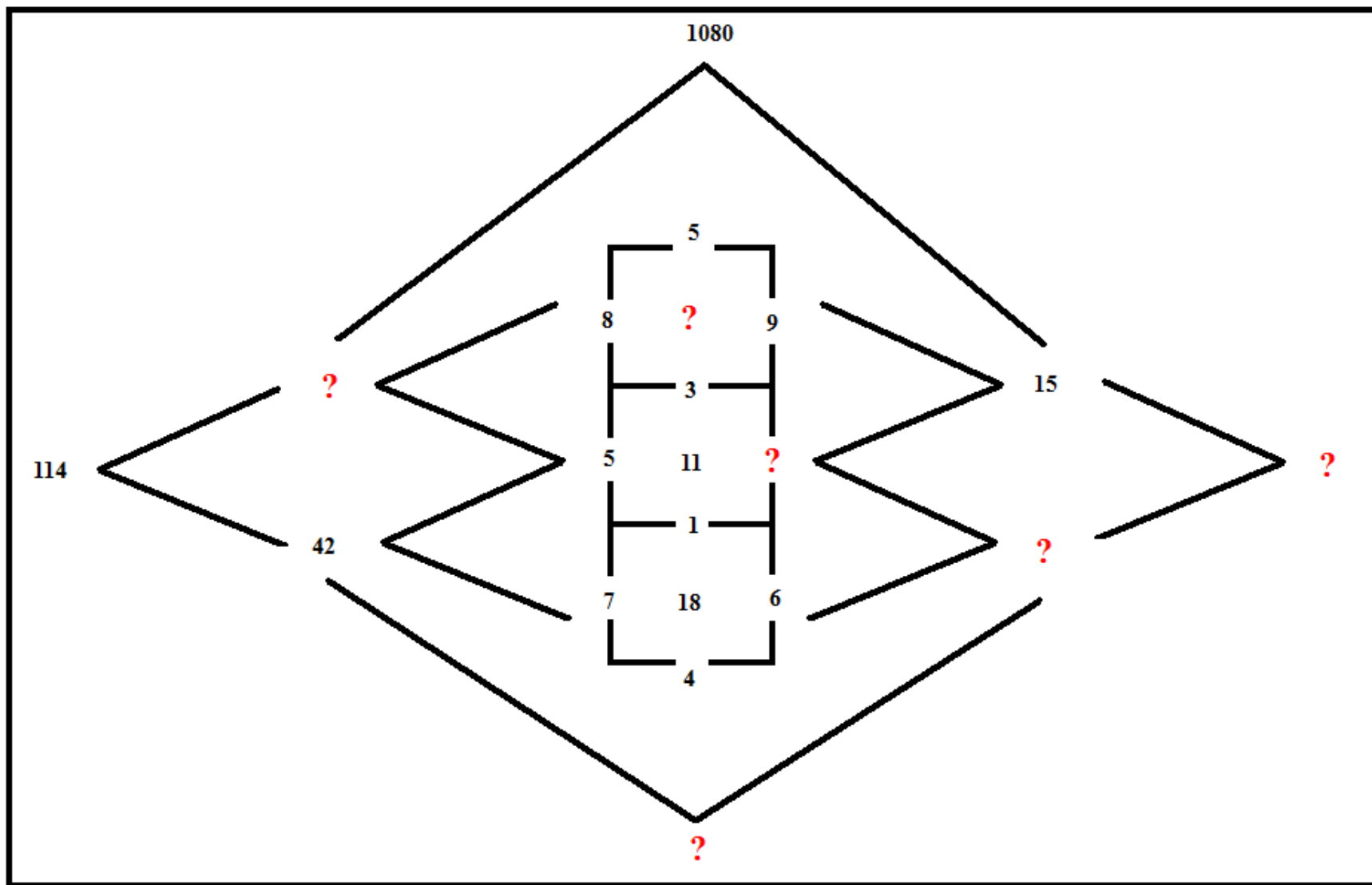
<sup>7</sup> *The Web of Belief*, Quine and Ullian, 1978. Chapter 2.

$\frac{A : B}{C}$  This is read: "If A is provable and it is consistent to assume B, then conclude C."

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**§5.0: Sequential Logical Reconstruction**

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In the previous figure, identify each of the missing numerical values.

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**§5.1:****Sequential Logical Reconstruction**

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Necessary Conceptual Information Prior to GameplayReconstructing Complex Epistemic Relations:

- $K_X\Delta$  means: X knows a state of affairs "delta" (simply stated: X whoever X is knows some state of affairs, classified as "delta").
- $K_R\Delta$  means: Donald Rumsfeld knows some state of affairs, classified as "delta".

Relational Epistemic Knowledge:

- $K_{XY}$  : this epistemic relationship is read as X knows Y.
- We can substitute X and Y with proper name, e.g.,  $K_{RB}$ , where R stands for Rumsfeld and B stands for Bob, therefore:  
 $K_{RB}$  is read as Rumsfeld *knows* Bob.

Symbolizing Relational Epistemic Knowledge and Testimony:**Non-relational:**

Where R: stands for Rumsfeld and B: stands for Bob

1.  $K_R\Delta$ : Rumsfeld knows some state of affairs "delta"
2.  $K_B\Delta$ : Bob knows some state of affairs "delta"
3.  $\sim K_R\Delta$ : Rumsfeld *does not* know some state of affairs "delta"
4.  $\sim K_B\Delta$ : Bob *does not* know some state of affairs "delta"

**Relational:**

From above. Here is the simple idea, before we complicate it with the logic. If 3 is true *and* 2 is *also* true then there *is* a means of transforming 3 *into* 1. Simply, Bob has to inform Rumsfeld about "delta," then Rumsfeld will know delta. I will demonstrate how that is done within the logic.

Ghetto Philosophy: I don't know your phone #. I want to know your phone #. I ask you your phone #. You tell me your phone #. I now know your phone #. It's that simple.

## Demonstration of Complex Relations:

K: to know

R: Rumsfeld

B: Bob

T: is to tell

$T_{RB}$ : this relation is read: "Rumsfeld tells Bob"

We can easily augment this by adding "delta" without disturbing the logical relation at all.

$T_{RB\Delta}$ : this relation is read: "Rumsfeld tells Bob about Delta"

The question then becomes, incorporating relations into our symbolization, how do we express the relational *transition* in knowing?

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$$T_{BR\Delta} \rightarrow [K_B\Delta \cdot K_R\Delta]$$

**Read:** *If Bob tells Rumsfeld about "Delta" then Bob knows delta and Rumsfeld knows delta.*

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$$[\sim K_R\Delta \cdot K_B\Delta] \rightarrow \{T_{BR\Delta} \rightarrow [K_B\Delta \cdot K_R\Delta]\}$$

**Read:** *If Rumsfeld does not know "Delta" but Bob knows "Delta" then if Bob tells Rumsfeld about "Delta" then Bob knows "Delta" and Rumsfeld knows "Delta"*

What has been demonstrated in the conceptual *means* with which *new information* [in terms of *knowledge*] becomes *known*. Rumsfeld has gone from a state of not knowing  $\sim K_R\Delta$  to a state of *knowing*  $K_R\Delta$  because of Bob testimony  $T_{BR\Delta}$ . Thus,  $T_{BR\Delta}$  is both a *necessary and sufficient* condition for  $K_R\Delta$ . [**KEY**]

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With this information we can now construct a rather complex game where information is reconstructed based on given knowledge.  $K_B\Delta$  in the previous example would be an instance of *given knowledge*. Similar to the numerical prompts I gave you in section 5 of the analysis. This new relationship, rather than being built within the system, will be built in *real life relationship between people*, e.g.,  $T_{BR\Delta}$ . Bob and Rumsfeld

must have some form of relationship which serves as the *means* of communicating knowledge of  $\Delta$  to Rumsfeld.

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**Game:** [*Advanced*] Reconstructing Complex Epistemic Relations:

**Given General Relations:**

1. K: [ $K_{xy}$ : is read: *X knows Y*]
2. I: [ $I_{xyz}$ : is read: *X introduces Y to Z*].
3. T: [ $T_{xy}\Delta$ : is read: *X tells Y about  $\Delta$* ].

**Given Players:**

1. R: stands for **Rumsfeld**
2. B: stands for **Bob**
3. M: stands for **Mary**
4. F: stands for **Frank**

**Given Specific Relations:**

1.  $K_{RB}$
2.  $K_F\Delta$
3.  $K_{MF}$
4.  $\sim K_{RM}$
5.  $K_{BM}$

**Demonstrations:**

1. Demonstrate  $K_{RM}$  from the *given* information with either logical and/or common sense explanation and defense. [your answer can be symbolized or you can write out in word how you arrived at  $K_{RM}$ ].
2. Demonstrate  $K_R\Delta$  from the *given* information with either logical and/or common sense explanation and defense. [**Hint:** begin with  $K_{RM}$ ]. [*there is a possibility for variations in the formulization*]. [your answer can be symbolized or you can write out in word how you arrived at  $K_R\Delta$ ].